MAT 151
Fall 2009
Take Home Quiz #4

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You will be graded on the effort you put into this quiz, not the accuracy. You may seek help, work together, ask a tutor. I want you to understand this material.

1. The population of the state of Alpha is given by an exponential function A = A(t) which has the formula $A = 17(1.09)^t$. Here A is measured in millions, and t is time measured in years since 1990.

a. What was the population of Alpha in 1990?

$t=0 \qquad 17(1.09)^0 = 17$ million

b. What is the yearly growth factor for A?

1.09

c. What is the yearly percentage growth rate for A?

9%

Quarterly growth Rate

$(1.09)^{\frac{1}{4}} = 1.0217$

2.17%

d. What is the monthly percentage growth rate for A?

$(1.09)^{1/12} = 1.007 = .7\%$

$X^{12} = 1.09$

$(\ )^{\frac{1}{12}} (\ )^{\frac{1}{12}}$

e. The population of the state Beta grows by 6% per year. In 1990 the population was 7 million. Let B be the population (in millions) of Beta and let t be time, measured in years since 1990. Find a formula for the exponential function giving B as a function of t.

$6\% \uparrow = 1.06$

$7(1.06)^t \qquad t=0$ in 1990

2. One of the two tables below shows linear data and the other shows exponential data (In this problem, round the ratios to 2 decimal places.)

T	0	3	6	9	12	15
F(t)	8.5	10.53	12.56	14.59	16.62	18.65

a. Is this data for f linear? Explain why it is or isn't.

$10.53 - 8.5 = 2.03$
$12.56 - 10.53 = 2.03$
$14.59 - 12.56 = 2.03$

constant rate of change, so the data is linear.

b. If it is linear, give a formula for f.

$y = mx + b$ $\quad y = \frac{2.03}{3}x + 8.5 = \boxed{y = .68x + 8.5}$

c. Is the data for f exponential? Explain why it is or isn't.

NO, The rate of change is constant. So it is linear.

$\frac{10.53}{8.5} = 1.24$
$\frac{12.56}{10.53} = 1.19$

Not exponential because we are not mult. by the same value each time.

d. If it is exponential, give a formula for f.

N/A

T	0	3	6	9	12	15
g(t)	8.7	10.09	11.71	13.58	15.75	18.27

a. Is this data for g linear? Explain why it is or isn't.

$10.09 - 8.7 = 1.39$ $\quad 11.71 - 10.09 = 1.62$

No, not a constant rate of change

b. If it is linear, give a formula for g.

N/A

c. Is the data for g exponential? Explain why it is or isn't.

$\frac{10.09}{8.7} = 1.16$
$\frac{11.71}{10.09} = 1.16$
$\frac{13.58}{11.71} = 1.16$

YES, The quotients are all equal (same multiplier) (mult. by the same value each time)

d. If it is exponential, give a formula for g.

$(1.16)^{\frac{1}{3}} = 1.05$

$y = 8.7(1.05)^x$

used expres. on calc.

3. In 1980 the population of Mexico was about 67.38 million. For the years 1980 through 1985, the population grew at a rate of about 2.6% per year.

a. Find the formula for an exponential function that gives the population of Mexico. Let t = 0 in 1980.

$100\% + 2.6\% = 102.6\% = 1.026$

$67.38(1.026)^t = P(t)$

b. Express using functional notation the population of Mexico in 1983, and then calculate that value.

$t = 3$

$67.38(1.026)^3 = 72.77$ million

c. Use the function you found in part a to predict when the population of Mexico will reach 90 million.

$67.38(1.026)^t = 90$

$1.026^t = \frac{90}{67.38}$

$\log(1.026)^t = \log\left(\frac{90}{67.38}\right)$

$t\log(1.026) = \log\left(\frac{90}{67.38}\right)$

$t = \frac{\log(90 \div 67.38)}{\log(1.026)} = 11.28$

$1980 + 11.28$

In 1991

4. Acidity of a solution is determined by the concentration H of hydrogen ions in the solution (measured in moles per liter of solution). Chemists use the negative of the logarithm of the concentration of hydrogen to define the pH scale:

$PH = -\log H.$

Lower pH values indicate a more acidic solution.

a. Normal rain has a pH value of 5.6. Rain in the eastern United States often has a pH level of 3.8. How much more acidic is this than normal rain?

$5.6 = -\log H$

$\log H = -5.6$

$10^{-5.6} = H_{normal}$

$3.8 = -\log H$

$\log H = -3.8$

$10^{-3.8} = H_{East\ U.S.}$

$\frac{10^{-3.8}}{10^{-5.6}} = 63.096$

$-3.8 + 5.6$

$10^{1.8} = 10^{\square}$

Rain in the Eastern U.S. is 63.096 times as acidic as normal Rain.

b. If the pH of water in a lake falls below a value of 5, fish often fail to reproduce. How much more acidic is this than normal water with a pH of 5.6?

$\log H = -5.6$

$10^{-5.6}$

$\log H = -5$

$10^{-5} = H$

$\frac{10^{-5}}{10^{-5.6}} = 10^{-5 + 5.6}$

$= 10^{.6} = 3.981$

This is 3.981 times as acidic as normal water.

$\log MN = \log M + \log N$

$\log \frac{M}{N} = \log M - \log N$

$\log m^n = n \log M$

5. Suppose $\log K = 4.1$

a. What is the value of $\log(3K)$?

$\log 3 + \log K$

$.477 + 4.1 = 4.577$

$\log_{10} K = 4.1$

$10^{4.1} = K$

b. If $\log L = 7$, how do K and L compare?

$L = 10^7$ $\quad K = 10^{4.1}$ $\quad L$ is $\frac{10^7}{10^{4.1}}$ times as big

794 times as big.

c. Find the value of K.

$10^{4.1} = K$

12589.254

6. Let $\log x = 2.1$. Find the following.

a. $\log 10x$

$\log 10 + \log x$

$1 + 2.1 = 3.1$

b. $\log \frac{x}{10}$

$\log x - \log 10$

$2.1 - 1 = 1.1$

7. Solve each equation.

a. $\log_4 8 = a$

$4^a = 8$

$\frac{\log 8}{\log 4} = 1.5$

b. $\log_3(b-2) = 4$

$3^4 = b-2$

$81 = b-2$

$b = 83$

c. $5(2)^{c+1} = 15$

$2^{c+1} = 3$

$\log_2 3 = c+1$

$\frac{\log 3}{\log 2} = c+1$

$\log 2^{(c+1)} = \log 3$

$(c+1)\log 2 = \log 3$

$c+1 = \frac{\log 3}{\log 2}$

$c+1 = 1.585$

$c = .585$

d. $2 \ln d = \ln 16 + \ln 4$

$\ln d^2 = \ln 64$

$\sqrt{d^2} = \sqrt{64}$

$d = \pm 8 = 8$